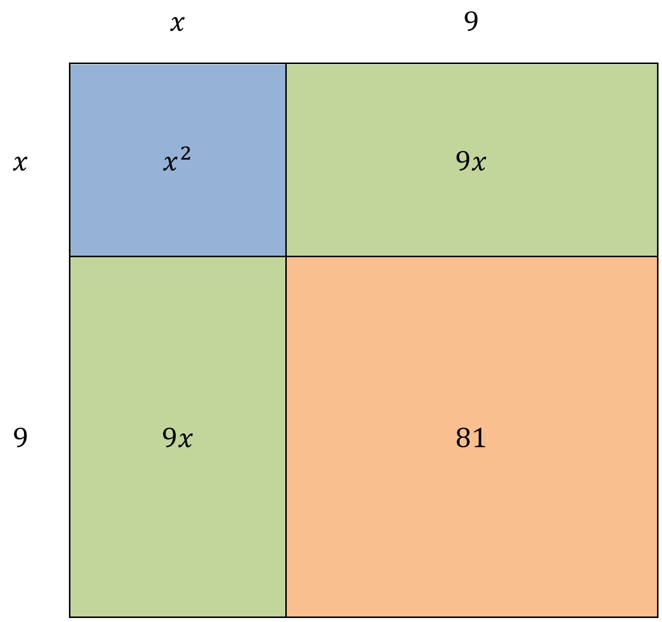
Perfect Square Trinomials

UNIT 7A: QUADRATIC EQUATIONS | PROBLEM SET 7A 02

The left-hand side of the standard form of a quadratic equation, $ax^2 + bx + c$, is called a **trinomial**. Trinomials that can be factored into two identical factors are called **perfect square trinomials**. One example of a perfect square trinomial is $x^2 + 18x + 81$ which can be factored into $(x + 9)(x + 9)$ or $(x + 9)^2$. The picture below helps us understand why this trinomial is called a “perfect square.”



Several other examples of perfect square trinomials are shown below:

$$x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$$

$$x^2 + 18x + 81 = (x + 9)(x + 9) = (x + 9)^2$$

$$9x^2 + 24x + 16 = (3x + 4)(3x + 4) = (3x + 4)^2$$

$$x^2 - 5x + \frac{25}{4} = \left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = \left(x - \frac{5}{2}\right)^2$$

$$4x^2 - 6x + \frac{9}{4} = \left(2x - \frac{3}{2}\right)\left(2x - \frac{3}{2}\right) = \left(2x - \frac{3}{2}\right)^2$$

Expand each product into a perfect square trinomial.

[ex] $(2x + 7)(2x + 7) = 4x^2 + 28x + 49$

[01] $(x - 5)(x - 5) =$

[02] $(x + 8)^2 =$

[03] $(4x - 3)^2 =$

[04] $(x - 1)(x - 1) =$

Factor each perfect square trinomial into a product of two identical factors to show that the trinomial is equal to a single factor squared. BEWARE, one of the trinomials in this set is NOT a perfect square trinomial. If you find it, don't try to factor it, just write "not a perfect square."

[ex] $x^2 + 12x + 36 = (x + 6)(x + 6) = (x + 6)^2$

[05] $x^2 + 16x + 64 =$

[06] $4x^2 + 12x + 9 =$

[07] $x^2 - 24x + 121 =$

[08] $9x^2 - 6x + 1 =$

[09] $x^2 + 20x + 100 =$

[10] $16x^2 + 40x + 25 =$

Let's give some special attention to those perfect square trinomials that begin with x^2, that is trinomials of the kind $ax^2 + bx + c$ where $a = 1$. Factor each of the following perfect square trinomials.

[11] $x^2 \mathbf{- 22}x \mathbf{+ 121} =$

[12] $x^2 \mathbf{+ 14}x \mathbf{+ 49} =$

[13] $x^2 \mathbf{- 30}x \mathbf{+ 225} =$

[14] $x^2 \mathbf{+ 4}x \mathbf{+ 4} =$

[15] $x^2 \mathbf{+ 32}x \mathbf{+ 256} =$

[16] What do you notice about the constant in **red** (the value of c) and the coefficient of x in **blue** (the value of b)? Is there any way to use the value of one to find the value of the other? Can you use a picture of some kind to justify any of your observations or claims?

Solve these quadratic equations. Let your first move be a factoring of the perfect square trinomial on the left side of the equation.

[ex] $4x^2 - 24x + 9 = 18$

$(2x+3)(2x+3) = 18$

$(2x+3)^2 = 18$

$2x + 3 = \pm\sqrt{18}$

$2x = -3 \pm 3\sqrt{2}$

$x = \dfrac{-3 \pm 3\sqrt{2}}{2}$

[ex] $x^2 + 18x + 81 = 0$

$(x+9)(x+9) = 0$

$(x+9)^2 = 0$

$x + 9 = 0$

$x = -9$

[17] $x^2 + 16x + 64 = 12$

[18] $4x^2 + 12x + 9 = 25$

[19] $x^2 - 24x + 121 = 27$

[20] $9x^2 - 6x + 1 = 3$

[21] $x^2 + 20x + 100 = 18$

[22] $16x^2 + 40x + 25 = 34$

[23] $x^2 - 22x + 121 = 75$

[24] $x^2 + 14x + 49 = 60$